CONSTRUCTING COMPLEXITY FOR DIFFERENTIATED LEARNING

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You can differentiate tasks to challenge learners at different levels of readiness. Develop alternate versions to increase or decrease task complexity to meet learners' needs.

Mrs. Cole has been teaching sixth- and seventh-grade mathematics for seventeen years and is respected by her colleagues. She excels at helping students achieve benchmarks and prepare for more advanced work. She has always worked hard to respond to her students' needs, usually teaching classes that are designated at particular levels relative to students' readiness and prior achievement. In recent years, her school has been moving toward more heterogeneous grouping of students. Mrs. Cole and her colleagues are facing an increasingly diverse range of readiness within each individual class. In planning each day's lesson, she strives to find ways to make the learning tasks respectful, accessible, and challenging for all her students without creating multiple lessons that occur simultaneously.

BEING MINDFUL OF INDIVIDUAL DIFFERENCES

Differentiated instruction requires that teachers pay attention to many differences among students, including their varied interests, learning styles, and ability and achievement levels. When responding to readiness, in particular, teachers must ask themselves many questions about the challenge levels of the learning tasks:

1. What makes a mathematics task easy or hard, and for whom?
2. How can we best provide support for students who are struggling while challenging those who are more advanced?
3. How can teachers integrate effective differentiation despite limited planning time?

This article grew out of a study that explored how the challenge level of learning tasks is connected to behavior for students who show strong potential in mathematics but are frequently off task (Simonsen, Little, and Fairbanks 2008). The idea is that when given appropriately challenging tasks, students may be more engaged and less likely to be off task or disruptive.

During the study, which was conducted in sixth- and eighth-grade mathematics classes, teachers gave us their planned independent practice activities for each day. Starting from these problem sets, we created sets of "hard tasks" and "easy tasks" and observed students at work. An important outcome of this process was extensive conversation about what defined a task as "hard" or "easy" and how this work related to the work of a teacher trying to provide differentiated instruction.

Differentiated instruction grows from the recognition that students learn at different paces and find challenge and stimulation in different types of tasks. Teachers using differentiated instruction hold high expectations for all students, using ongoing assessment to guide instructional decision making. They begin with worthwhile objectives and strong curricular materials, then use evidence of students' needs to provide a variety of learning experiences. Through these experiences, the hope is that students develop understanding and can demonstrate what they have learned.

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Sometimes this variety is designed to appeal to students' interests, respond to their diverse learning preferences and profiles, and ensure challenge for their varied readiness levels. Often, of course, activities might respond to more than one area of student difference. For example, a teacher might present a task that allows students to choose from a menu of topics and a list of product formats; or a teacher might distribute two versions of a task, at different levels of complexity, but then allow students within each task group to choose whether to work independently or with a partner. In each case, the objectives and tasks for all students should reflect best practices in the content area. However, differentiated tasks should allow students to engage with learning in ways that best meet their own needs.

Differentiation relies on flexible grouping, clear expectations, and a shared understanding that different students might be doing different things at the same time. In a mathematics classroom, whole-class instruction might be followed by independent practice using problem sets that are at different levels of complexity. Or a teacher might begin by distributing varied problem sets and then spend the class rotating from one small group to another to provide targeted instruction. These groups change from day to day and unit to unit, according to student needs and interests, as demonstrated through ongoing assessment.

USING TIERING AS A STRATEGY

A number of strategies can be used to streamline planning for differentiation and integrating it into instructional practice. Designing learning and interest centers, using anchor activities, producing menus of options to allow for student choice, and establishing learning contracts with individual students are just a few approaches.

One important strategy for differentiation is *tiering*. It involves preparing multiple tasks or versions of tasks that respond to common objectives while providing variety in their levels of complexity and challenge, the learning styles they address, or the interests to which they appeal (Heacox 2002; Pierce and Adams 2005).

Often, tiering is used to refer to tasks that are differentiated to respond to differing levels of readiness, with a focus on the level of challenge presented by the tasks (Tomlinson 2003). Through tiering, mathematics teachers can give all students challenging tasks while ensuring sufficient scaffolding for struggling students and reducing repetition for more advanced students.

Although tiering for readiness involves adjusting tasks to fit students' varying ability levels, it is done with an eye to ensuring that all students are presented with respectful and worthwhile mathematical tasks. Such tasks offer students opportunities to draw on their past experiences, use significant mathematics, and understand and make connections across mathematical ideas (Carpenter et al. 1997; NCTM 2000).

The degree to which the tasks are respectful for individual students and groups of students depends on the adjustments made for student readiness. One key principle of a differentiated classroom is that students have a right to " 'begin where they are' and to expect to grow as learners" (Tomlinson 2001, p. 93). Therefore, although a teacher's initial development or selection of a worthwhile task is grounded in representing important mathematical skills and ideas, the adjustments depend on recognizing student areas of strength and need through ongoing assessment.

Tiering for readiness begins with selecting or developing a worthwhile learning task that will help students achieve one or more meaningful objectives. Then a teacher develops several adjusted versions of the task, with the same or similar objectives, based on students' assessed needs (Tomlinson 2003). For example, some students may need assignments that have more complexity or abstraction; others might need more concrete tasks. Developing the tiers may involve constructing new problems or carefully selecting and distributing existing problems. By selecting or designing tiers around the same core mathematical ideas and processes and by starting from a worthwhile task, teachers can also then assess all students' progress toward the core objectives.

It is critical in this context to recognize key distinctions between assessment and grading. Tiering depends on assessment, because it requires gathering information about student progress to make instructional decisions. Performance on a preassessment or an in-class activity one day will help a teacher decide which level of a tiered assignment a student should receive the next day. A student who is struggling with a concept during a whole-class discussion—or finding it not very challenging—may be assigned to a particular tier for that day on the basis of the informal assessment of discussion participation. Although these instructional decisions often result in tasks that vary in overall expectation from one student to another, in a well-designed tiered assignment all students will be working toward the same objectives at levels that are respectful of their individual needs. The experience will also allow students to grow in their learning.

The value of differentiation to respond to student readiness emerges from the opportunities for all students to work with tasks that challenge them.

Grading, on the other hand, represents a reporting of student achievement, usually at an end point. Literature on differentiation strongly supports the development of grading systems that combine attention to achievement of the objectives with a consideration of progress and effort. The literature also supports including these components within reporting systems (e.g., Tomlinson and McTighe 2006; Wormeli 2006). In addition, these authors recommend that not everything students do be graded.

Performance on a tiered assignment may be considered as part of the overall evidence of progress but need not be formally graded except to indicate satisfactory or unsatisfactory progress. A full discussion of grading within a differentiated classroom is not included here, but interested readers are encouraged to look to the references for more information.

To establish multiple tiers, a teacher must consider what characteristics make a task more or less challenging for given learners. Tomlinson (2001, 2003) provides some guidance by suggesting multiple ways of adjusting tasks. She identifies a number of continua that describe learning tasks and explains that differentiation occurs through adjusting where a task falls on one or more of these continua. For example, tasks may be defined on a continuum from concrete to abstract; in a simple example, a given problem may be made more concrete by giving students manipulatives. Tomlinson has compared these different qualities of learning tasks to the adjustable buttons on a stereo. She calls her metaphor the "equalizer" and explains that multiple "buttons" may be adjusted on any given task to create the right "sound" for a given student or group.

In the following sections, we explore several of the specific equalizer continua in more detail, with specific problems that illustrate how they might be represented in mathematics tasks at the middle school level.

INCREASING OR DECREASING THE FACETS OF A TASK

One straightforward way to differentiate tasks is to increase or decrease the number of facets in a problem, thereby adjusting its complexity. Examples of facets that can be adjusted in a mathematics problem include the number of steps required to solve it, the number of variables, or the number of different skills to be employed.

In general, problems may be differentiated for more advanced or

Fig. 1 Factoring tasks that provide different facets. The trinomials in (b) contain additional facets resulting from the composite leading coefficient.

(a) Factor each trinomial.

$3x^2 + 14x + 8$ $\quad$ $2x^2 + 7x + 3$ $\quad$ $x^2 - 2x - 3$

(b) Factor each trinomial.

$6x^2 + 28x + 16$ $\quad$ $6x^2 + 21x + 9$ $\quad$ $2x^2 + 4x - 6$

Fig. 2 A statistics task that can be tiered for different students could include, for example, simpler number combinations.

Jill and her five friends love to watch movies. One week, each person watched at least 1 movie, and the mean of the six values was 7 movies. Find a set of six amounts of movies watched that have a mean of 7 movies.

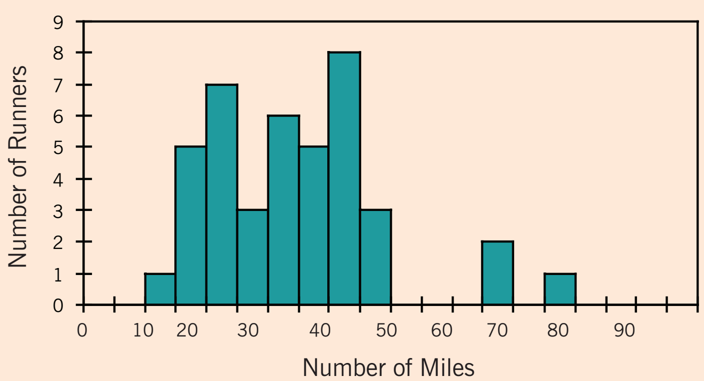
Is there more than one set of six amounts that has a mean of 7 movies? If so, list another set, and explain how you knew another set was possible. If not, explain how you know.

(*Note:* This task was inspired by a problem in Bright et al. 2003.)

Fig. 3 This task can be used on many levels to enforce histogram-reading skills or foster transformational thinking.

The histogram below shows the distances run in one week by runners training for a marathon. These data were reported in whole numbers of miles.

Distances Run by Marathon Runners



The mean and median of the individual data values are 30 and 32 but not necessarily in that order. Which value is the mean? Which value is the median? How were you able to determine this?

(*Note:* This task was inspired by a problem in Bright et al. 2003.)

struggling students by increasing or decreasing the number of facets. This is distinctly different from giving students more or fewer total problems. Instead, the focus is on the internal components of the tasks. For those who are struggling to grasp a concept, operation, or application, teachers may decrease facets by reducing the number of steps or other components of a problem that may go beyond the central objective.

Consider the sets of problems in **figure 1**. Both sets ask students to factor trinomials, but each trinomial in (b) includes a facet that is missing from (a). A teacher could give (a) and (b) to different groups of students; for example, (a) could be assigned to those who might be struggling to grasp the concept. Although teachers would want all their students eventually to complete problems such as those in (b), some students will be ready sooner than others.

Observing how students respond to the tasks in (a) will give teachers a basis for assigning further practice problems such as those in the task or for moving a student on to problems like those in (b). A student's performance on the problems in (b) will help the teacher determine whether he or she needs to try a few practice problems resembling those in (a), continue with problems similar to those in (b), or move on to a new topic or skill.

Consider the problem that is in **figure 2**. To reduce the number of facets, a teacher might present simpler number combinations, such as four values with a mean of 6 or three values with a mean of 7. The key concepts of the problem—finding mean values and recognizing that different data sets can produce the same mean—remain the same, but students are asked to grapple with fewer different values and thus fewer possible distractions from the key concepts.

As an alternative, teachers may choose to provide extra scaffolding on a problem, adding guidance to the steps or components involved. This approach does not directly decrease the number of facets; rather, it limits the number of facets a student has to handle at a time. For example, the trinomial problem set (b) in **figure 1** might be adjusted by adding the direction to check for a common factor.

For more advanced learners, adjusting the number of facets might involve adding extra steps or variables or requiring a greater range of skills. The problem in **figure 3** could be adjusted for more advanced students as follows:

> The mean, median, and mode of the individual data values are 32, 34, and 44 but not necessarily in that order. Which value is the mean? Which is the median? Which is the mode? How are you able to determine this?

This information increases complexity by asking students to consider more variables at one time. Advanced learners often need only a few practice problems to master an objective. Therefore, adding more facets through extension questions can provide an appropriate challenge, as in this question that extends **figure 2**:

> One week, Jill noticed that each person watched an odd number of movies, no two people watched the same number of movies, and the mean was 6 movies. Find a set of six values that satisfies these conditions.

Such an extension problem might take the place of additional practice problems for those students who are struggling with the concept.

MOVING ALONG THE CONTINUUM

Altering the level of abstraction is another way to adjust the complexity of a task. Using more variables and generalizations involves a greater focus on meanings and connections. As students move from concrete numbers into variables, a higher level of abstraction is introduced. In the more advanced courses of algebra, geometry, and trigonometry in high school, teachers present material that is naturally more abstract than much of what students have previously experienced. Thus, it is helpful for middle school students to grapple with problems that introduce increasing layers that balance concreteness and abstraction adjusted for student readiness.

Explaining reasoning, making connections, and exploring predictions are important mathematical skills for algebra and beyond, but these skills need to be fostered much earlier and particularly reinforced in middle school. For example, consider the following extension for the task presented in **figure 2**:

> If the mean number of movies watched in a week is 8, and if every girl watches a different number of movies, what is the smallest possible range for the set of six values? Explain your answer.

This problem adds a level of reasoning and connection. Although the original question asks students to explore the basic concept of mean, this problem adds a level of abstraction because there is not one clearly specified way to approach the problem. Students also have to think abstractly about the number set in the problem and be able to explain their thinking.

Many middle schools offer algebra and even geometry for more advanced students. With the greater abstraction

Fig. 4 Two degrees of reasoning with algebraic tasks

(a) Factor each expression.

$x^2 - 4x + 4$	$12x^2 + 7x + 1$	$4x^2 - 16$

(b) Factor each expression if possible. If it cannot be factored, write “not factorable” and explain your answer.

$x^2 - 4x + 4$	$12x^2 + x + 8$	$4x^2 + 16$

Key Ideas for Successful Differentiation

The following ideas will help you provide differentiated instruction for all students:

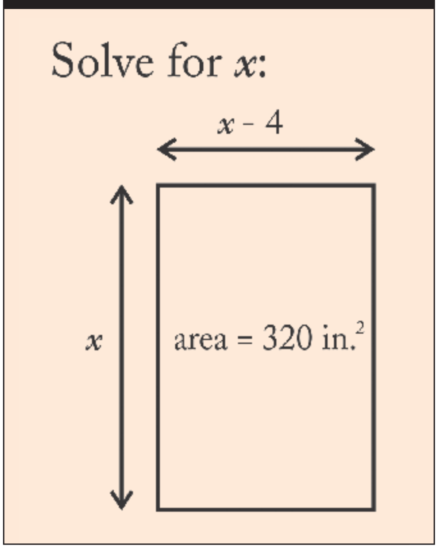
- Start small. Use materials that you are already working with and adjust them to respond to varied needs around your objectives.
- Promote growth for all learners; keep struggling, grade-level, and advanced students in mind.
- Give all students access to rich, worthwhile tasks and ideas that encourage higher-level thinking and mathematical applications.
- Adjust the number of tasks along with the complexity, but avoid giving any group of students significantly more or fewer problems to solve.
- Use assessment continuously, and group flexibly according to assessed needs.
- Recognize that some students may have needs beyond what can be met with tiering.

inherent in the content, increasing the abstraction further can provide additional differentiation for some students. Consider the sets of tasks in **figure 4**. The problems in (b) increase abstraction by including items that cannot be factored. Only the first expression is actually factorable into binomials, although the third expression has 4 as a common factor. Students' responses to the second and third items would be important indications of their understanding of the concepts, based on the degree to which they can explain why they cannot be factored.

On the third item, a student could provide a correct response with just 4 as a common factor; another student might add an explanation as to why the expression cannot be factored further into binomials. This response demonstrates more thoughtful reasoning; it would give teachers evidence for determining if the student needed more practice on similar problems or encouragement to explain additional thinking on this task. The response would also tell the teacher if the student was ready to move on.

The (b) tasks in **figure 4** require students to demonstrate more understanding of the concepts and communicate reasoning, moving them toward the more advanced skills of proof. At the middle school level, it is more appropriate to use reasoning rather than formal proof (Gray et al. 1999). All students should be asked to demonstrate their reasoning in mathematics. However, by increasing the emphasis on reasoning, a teacher may also increase the abstraction required by the task for more advanced students.

Fig. 5 A task that asks students to create their own equation requires transformational thinking.



Through tiering, mathematics teachers can give all students challenging tasks while ensuring sufficient scaffolding for struggling students and reducing repetition for more advanced students.

EXPANDING PROBLEMS

Another way to differentiate tasks is to adjust the degree to which they ask students to stretch their understanding. In so doing, students will be moving from the foundational skills of a discipline to transformational applications of these skills in new contexts. When students are first presented with a concept or procedure or are having difficulty grasping it, their assigned tasks should be designed to build understanding and establish a strong foundation. Foundational tasks in mathematics typically present material in a similar context to how it was taught and require practice of basic skills. For example, the following problem presents a straightforward equation:

Solve for x: $320 = x(x - 4)$

As students develop greater understanding of a concept, they may be ready to use what they have learned in a different context to find new connections or think about the concept in a new way, which in its entirety can be regarded as *transformational thinking*. For example, students might be given a task such as that in **figure 5**.

This task is more transformational because it asks students to make connections and put the equation together themselves. It often requires students to apply concepts and skills to real-life contexts and problems in ways that they have not been explicitly taught. The following task moves the problem still further along the continuum toward transformational:

> You are putting a new in-ground pool in your yard. The area of the pool is 320 m^2, and the pool is 4 m longer than it is wide. You want to put a patio 2 m wide all the way around the pool. What will be the total area of your patio?

This problem asks students to think about the concepts in a new context as they devise a solution method. Such problems reflect the practices of mathematicians in applying key ideas and principles to new and unsolved problems and to real life.

Transformational tasks ask students to demonstrate a more thorough understanding of key concepts. For example, the statistics problem in **figure 2** is transformational in that it requires a greater understanding of the mean than a problem presenting individual data points.

Sometimes, a teacher's goal might be to give all students greater access to a transformational task. One way

Fig. 6 Three levels of tasks working with linear equations

Level 1	Level 2	Level 3
Linear Equations in Two Variables For the equations below, tell whether the ordered pair is a solution to the given equation. Show why or why not. **1.** $y = x + 2$; (2, 4) **2.** $y = -2x + 7$; (–3, 1) **3.** $x - 4y = 3$; (7, 1) **4.** $3x + 2y = -4$; (–3, 2)	**Linear Equations in Two Variables** Find the value of *b* that makes the ordered pair a solution for the following equations. **1.** $y = x + 2$; $(2, b)$ **2.** $y = -2x + 7$; $(b, 1)$ **3.** $x - 4y = 3$; $(7, b)$ **4.** $3x + 2y = -4$; $(b, 2)$	**Linear Equations in Two Variables** Find the value of *b* that makes the ordered pair a solution for the following equations. **1.** $y = -2x + 7$; $(-b, 1)$ **2.** $x - 4y = 3$; $(7, b - 5)$ **3.** $3x + 2y = -4$; $(7 - b, 2)$
5. The formula $y \approx 2.6x$ changes the area *x* of a region in square miles to an approximate area *y* in square kilometers. The local wildlife preserve has an area of 768 square miles. Find the area of this preserve in square kilometers. Round your answer to the nearest square kilometer.	**5.** The formula $y \approx 2.6x$ changes the area *x* of a region in square miles to an approximate area *y* in square kilometers. The local wildlife preserve has an area of 1196 square kilometers. Find the area of this preserve in square miles. Round your answer to the nearest square mile.	**4.** The formula $y \approx 2.6x$ changes the area *x* of a region in square miles to an approximate area *y* in square kilometers. Write a formula for finding the number of square miles given the number of square kilometers. Use this new formula to find the area in square miles of a local wildlife preserve with an area of 1196 square kilometers. Round your answer to the nearest square mile.
Write the following equations in function form. Then solve for *y* in each problem if $x = 2$. **6.** $y - x = -4$ **7.** $4x + 2y = 3$ **8.** $x = 3y - 2$	Write the following equations in function form. For each equation, give two ordered pairs that would be solutions. **6.** $y - x = -4$ **7.** $4x + 2y = 3$ **8.** $x = 3y - 2$	Write the following equations in function form. For each equation, give two ordered pairs that would be solutions. **5.** $4x + 2y = 3$ **6.** $x = 3y - 2$ **7.** Explain how it is possible to have more than one ordered-pair solution.
Level 1	Level 2	Level 3

is to break it into smaller tasks, which illuminate the more foundational steps needed to solve a more complex task. To make **figure 2**'s problem more accessible, a teacher might first give students two data sets of six values and a mean of 7 and ask them to find the mean and explain how it is possible to have two different sets with the same mean. Then, the teacher can guide the students to find a third set with the same mean.

Existing foundational tasks can also be adjusted to make them more transformational, perhaps by asking students to generate their own data to meet certain constraints, make judgments, or work backward. Transformational tasks may also be presented through extension problems, such as this example involving **figure 3**:

> On grid paper, create a new version of the histogram with 10 miles as the interval for each bar. What information is lost or gained? Can you answer the same questions from the new histogram? Why or why not?

If some students who are given **figure 3** need more practice interpreting histograms, this extension would be a reasonable activity for more advanced students in place of working on another foundational problem.

PUTTING THE PIECES TOGETHER

The sections above have highlighted ways to differentiate individual tasks through adjustments to particular problem characteristics. As teachers put these ideas together, one practical approach is to develop overlapping independent practice activities. This might mean presenting one challenging stimulus to all students, with different tiers adjusting levels of complexity. For example, **figure 3** might be presented to all students. Some could be asked more basic histogram questions, whereas others could complete more challenging extension problems.

Overlapping problem sets might also include several activity sheets with a few common problems and some unique problems. The three examples in **figure 6** increase in complexity from level 1 to level 3. Each of the continua addressed above is reflected in the distinctions among the three. For example, the first section of problems moves from more concrete to more abstract across levels with the addition of a variable.

The word problem gives all students a related task but with more facets to address as levels increase. The third section moves toward transformational, particularly with the inclusion of the last question, which requires a more complete understanding of what linear equations represent.

CONCLUSION

The value of differentiation to respond to student readiness emerges from the opportunities for all students to work with tasks that challenge them. By analyzing activities carefully for their degrees of complexity and challenge, teachers can more readily identify appropriate tasks for students with different needs. They can then structure versions of tasks that are coordinated and stimulating across multiple levels. Thus, individual students will be challenged where they are.

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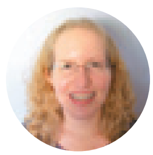


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